Department of Education and Early Childhood Development

Key Characteristics of Effective Numeracy Teaching 7-10

November 2009

Differentiating support for all students

Published for Student Learning Division
Department of Education and
Early Childhood Development
Melbourne
September 2009

Authorised by the Department of Education
and Early Childhood Development,
2 Treasury Place, East Melbourne, Victoria, 3002.

This document is also available on the internet at
http://www.education.vic.gov.au/studentlearning/teachingresources/preptoyear10.htm

Contents

Introduction

The international evidence-base has reached consensus about the significant influence teachers have in the classroom and the knowledge and the skill base required to teach all students well. In order to improve teacher practice, teachers must not only understand what it looks like to improve in different domains of learning but they must also believe they are capable of improving their practice.
(e^5 Instructional Model, DEECD, 2009)

Students within each class present their teachers with a wide range of mathematics experiences, knowledge and skills. This provides a significant challenge as teachers need to ensure that all students are provided with the knowledge and skills to continually improve their mathematics learning.

International research indicates that to respond effectively to the diversity of student learning needs and aspirations at all stages of learning, teachers need to consider systematically 'two phases of teaching':

- The **first** phase–quality differentiated classroom teaching for all students.
- The **second** phase–additional short-term intervention for students not achieving the expected level in numeracy in order to accelerate their learning.

The Key Characteristics of Effective Numeracy Teaching 7-10 specifies the nature of first phase high quality differentiated classroom teaching for all students. These characteristics were informed by research and developed through consultation with classroom practitioners, numeracy experts, coaches and regional personnel.

These key characteristics will support teachers of other domains to recognise and build on their students' numeracy skills and understanding within domains, allowing students to see the relevance of mathematics learning and develop deeper mathematical understanding.

This resource can be used by teachers, coaches and school leaders to identify:

- **Teacher knowledge**–disciplinary and pedagogical content knowledge that teachers require including VELS Mathematics domain, Mathematics Developmental Continuum P-10, and information on particular cohorts of students.
- **Numeracy focus**–advice on essential knowledge and skills that teachers need to focus on.
- **Assessment**–advice on key assessments and their timing to inform teaching including Fractions and Decimals Online Interviews: VCAA On Demand Linear Tests, including Progress Tests, and VCAA On Demand Adaptive Tests, Scaffolding Numeracy, NAPLAN tests.
- **Planning and instruction**–advice on organisational structures, recommended teaching strategies. This section also refers to the e^5 Instructional Model which provides a framework to support differentiated, purposeful teaching.

The document is structured for

- Years 7 and 8, and
- Years 9 and 10

In a differentiated classroom, the teacher proactively plans and carries out varied approaches to content, process, and product in anticipation of and response to student differences in readiness, interest, and learning needs.
(e^5 Instructional Model, DEECD, 2009)

Suggested Readings

- Mathematics domain page

The Mathematics domain page contains links to: Victorian Essential Learning Standards; mathematics-related learning and teaching support materials such as the Mathematics Developmental Continuum P–10, research, web sites, publications and other online resources; mathematics assessment maps and sample tasks; professional learning support including links to mathematics teachers' associations; and identifies current research in the area of mathematics education.

- Research eLert Numeracy in practice: teaching, learning and using mathematics

This report produced in 2009 focuses on research and links related to the characteristics of effective numeracy teaching and addresses key classroom issues including: what to teach, how to teach numeracy, how to cater for diversity and how to make best use of technology.

- Researching Numeracy Teaching Approaches in Primary Schools

The major outcome of this research undertaken from 2000-2003 in Victoria is the identification, description and elaboration of twelve scaffolding practices that contribute to improved student learning outcomes.

- Middle Years Numeracy Research Project (MYNRP)

The Middle Years Numeracy Research Project was commissioned to inform the development of a strategic and coordinated approach to the teaching and learning of numeracy for students in Years 5-9, the final report was published in 2001.

Years 7 and 8

Teacher knowledge

Effective mathematics teachers have:

- thorough knowledge of VELS mathematics domain learning focus statements and standards and progression points– levels 3, 4, 5 and 6
- knowledge of the Mathematics Developmental Continuum P-10 and knowledge of the links to all the indicators of progress within the Continuum across the dimensions
- knowledge of key mathematical concepts through all levels of learning Developmental Overviews
- knowledge of the Big ideas linked to the Fractions and Decimals Interview
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from Koorie, ESL and or Low SES backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective mathematics teachers:

- develop students' understanding of the meaning and use of digits, natural numbers, integers, rational numbers, the importance of the placement of zero especially in decimals and to identify complete factors sets for natural numbers and express natural numbers as products of powers of primes
- develop students' knowledge of and skill in using fractions and their reciprocals, given in simplest form ($\frac{1}{2} = \frac{5}{10} = \frac{15}{30}$) and knowing decimal equivalents for the unit fractions ($\frac{1}{4} = 0.25$, $\frac{1}{5} = 0.2$) and engage students in evaluating natural numbers and simple fractions given in base exponent form and calculating the equivalent decimals, ratios and percentages and using symbols to represent rational numbers
- use students' knowledge of perfect squares when calculating and estimating squares and square roots of numbers and cube and cube roots to a specified degree of accuracy and increasing their knowledge of the simple powers of 2, 3 and 5 and to generalise from perfect square and difference of two square number patterns
- engage students in identifying collections of numbers as subsets of integers, natural, rational and real numbers and teach them to identify number sets (empty, power and finite), list the elements of the power set (set of all subsets) of a given finite set and to comprehend the partial-order relationship between these subsets with respect to inclusion and develop students' understanding of ratios as set:set and subset:set comparisons and the use of diagrams and graphs to illustrate the relationships between sets (intersection, union, inclusion and complement)
- teach students to use variables in general mathematical statements and substitute numbers for variables and to identify the correspondence of a function between two sets (one-to-one or many-to-one) and to represent it by a table of values, a graph, and by a rule and to describe and specify the independent variable (and its domain) and the dependent variable (and its range) of a function and construct tables of values and graphs for linear functions and to model various situations
- teach students to apply number properties (commutative, associative, and distributive) in mental and written computations and use exponent laws for multiplication and division of power terms and to become fluent at recognising and manipulating symbols, formulae and algebraic expressions, solving simple equations using tables, graphs and inverse operations
- engage students in building efficient mental, written and technology based strategies for arithmetic computation and in using technology for a range of mathematical purposes

- introduce students to binary notation and the addition and subtraction of natural numbers in binary form
- teach students to identify parallel lines and use the transversals of these lines to calculate alternate, supplementary, corresponding and allied angles
- teach students about the properties of quadrilaterals and congruent and similar triangles and engage them in solving geometrical problems applying these properties and justifying their results and in explaining geometric propositions
- engage students in visualising and constructing simple 3-D objects from 2-D nets and in using single-point perspective to make a 2-D representation of a simple 3-D object
- teach students to recognise and apply simple geometric transformations of the number plane (translation, reflection, rotation and dilation) as well as combinations of these (including their inverses) and to use precise map references, contour lines, bearings and Cartesian coordinates and more complex map scales
- teach students to measure, using suitable units and estimate the accuracy of measurements, give suitable lower and upper bounds for measurement values in context and calculate absolute percentage error of estimated values
- engage them in using measurement formulas to calculate area and perimeter of circles, triangles and parallelograms and simple composite shapes and surface area and volume of prisms and cylinders
- teach students to identify empirical probability as long-run relative frequency and calculate theoretical probabilities by dividing the number of possible successful outcomes by the total number of possible outcomes and use tree diagrams to investigate the probability of outcomes in simple multiple event trials
- teach students to use appropriate technology to generate random numbers in the conduct of simple simulations and engage them in analysing the reasonableness of points of view, procedures and results according to given criteria, and identify limitations and/or constraints in context
- teach students to organise, tabulate and display discrete and continuous data using technology for larger data sets, to represent uni-variate data in appropriate graphical forms, to calculate summary statistics for measures of centre and make simple inferences based on this data
- teach students to test the validity of statements formed by the use of the connectives (*and, or, not*) and quantifiers (none, some and all) and to apply these to sets (with one and two attributes) and database searches
- teach students to formulate conjectures, follow simple mathematical deductions, develop simple mathematical models for real situations and develop generalisations by abstracting the features from situations, expressing these in words and symbols and to predict using interpolation and extrapolation.

For students achieving above the expected level, teachers should refer to the appropriate VELS standards and progression points

For students needing additional assistance, teachers should refer to earlier VELS standards and progression points

Assessment

Effective teachers continually monitor and track the progress of individual students.

At the **beginning of the year**, effective teachers:

- use information from transition statements and data including: VELS teacher judgements, VCAA on demand testing and year 7 NAPLAN results to understand the starting point for each student
- administer and analyse VCAA On Demand Adaptive Testing – a range of computer-based assessments which identify a student's achievement level. It is recommended that teachers use this tool to identify the spread of achievement within the class and then to use the progress test to gain more detailed information about individual students

administer and analyse Fractions and Decimals Online Interview

For students achieving below the expected level:

- administer the assessment materials to assess students' multiplicative thinking. Scaffolding Numeracy in the

Middle Years, analyse the assessment outcomes using Learning and Assessment Framework for Multiplicative Thinking and plan for future learning by using Learning plans.

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual students against VELS Mathematics standards and progression points
- administer and analyse VCAA On Demand Progress tests which are linear tests designed to measure outcomes against the VELS
- administer and analyse, for identified students, the Assessment for Common Misunderstandings. The tools comprise a number of easy to administer, practical assessment tasks designed to address key areas of Number.
    - (LEVEL 5 – Proportional reasoning, extending what is known about multiplication and division beyond rule-based procedures to solve problems involving fractions, decimals, percent, ratio, rate and proportion)
    - (LEVEL 4 – Partitioning, the missing link in building common fraction and decimal knowledge and confidence)
    - (LEVEL 3 – Multiplicative thinking, the key to understanding rational number and developing efficient mental and written computation strategies in later years)
- regularly observe students working mathematically across all dimensions within the class context, particularly during independent mathematics and document progress against the VELS standards and progression points
- provide students with regular opportunities for self assessment and self reflection
- use the VCAA mathematics assessment maps to help assess student work using the VELS to support consistent, on-balance judgments about student achievement.

At the **end of each semester**, effective teachers:

- integrate evidence collected throughout the semester to make on balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective mathematics teachers:

- demonstrate capabilities as described in the e^5 Instructional Model
- dedicate as a minimum four hours weekly to explicit mathematics teaching
- use a range of flexible student groupings ensuring appropriate level of differentiated teacher support: whole class focus, small groups, independent activities and whole class reflection and analysis
- organise learning spaces to enable differentiated teaching and collaborative learning
- plan and design sessions, making connections to mathematical ideas from previous sessions or experiences, different parts of mathematics or contexts
- structure purposeful, authentic mathematical tasks that engage students to envisage different possibilities, use a range of strategies allowing diverse products to emerge, and encourage higher order thinking skills
- develop student mathematical understanding through teacher strategic questioning and feedback and student explanation of reasoning and methods, as well as using multiple representations to make content accessible to all
- provide independent mathematics time to enable student to practise what they know and to act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by introducing new terms and symbols meaningfully and expect and encourage correct use, making connections between language, symbols and materials
- provide opportunities and resources that enable students to manipulate concrete materials to construct their own understandings and use these resources in the application of concepts and skills.

Recommended teaching strategies

Effective numeracy teachers:

- differentiate instruction according to student need
- explicitly teach students strategies to approach mathematical problems,
- select appropriate teaching strategies including Classroom Activities, following analysis of the Fractions and Decimal Online Interview
- engage students in discussion, reflection and active construction throughout sessions, to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of scaffolding practices to support teachers to make more informed decisions about how they will meet the learning needs of all students in the most appropriate way possible
- select from the twelve scaffolding practices and use appropriate to purpose throughout the five phases of instruction as described in the e^5 Instructional Model.

Considerations for materials selection

Effective mathematics teachers select a range of materials that

- are based on the students' developmental stage and the mathematics being explored
- are appropriate to the learning context and are socially and culturally inclusive.

Years 9 and 10

Teacher knowledge

Effective mathematics teachers have:

- thorough knowledge of VELS mathematics domain learning focus statements and standards and progression points – levels 4, 5 and 6
- knowledge of the Mathematics Developmental Continuum P-10 and knowledge of the links to all the indicators of progress within the Continuum across the dimensions
- knowledge of key mathematical concepts through all levels of learning Developmental Overviews
- knowledge of the Big ideas linked to the Fractions and Decimals Interview
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from Koorie, ESL and or Low SES backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective mathematics teachers:

- develop students' understanding of the set of real numbers (natural, integer, rational and irrational) and teach them to classify and describe the properties of the real number system and the subsets of rational and irrational numbers; identifying subsets as discrete/continuous, finite/infinite, providing examples of their elements and applying these to functions, relations and the solution of related equations
- build students' skills at performing computations, using both mental and written algorithms, involving natural numbers, integers and finite decimals, fractions and irrational numbers such as square roots and solving percentage problems such as adding and taking off a percentage and solving ratio and proportion problems in a variety of ways
- engage students in using appropriate estimates to evaluate the reasonableness of the results of calculations involving rational and irrational numbers, and the decimal approximations for them and extend their skills at forming and testing mathematical conjectures
- teach students to apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computations with number, to rearrange formulas and simplify and verify the equivalence or otherwise of algebraic expressions involving real variables (linear, square, cube, exponent, and reciprocal)
- engage students in representing rational numbers as fractions and decimals and specifying decimal approximations for the square root of primes, rational numbers that are not perfect squares, the golden square θ and simple fractions of π to a required decimal place accuracy
- teach students to simplify surds and rationalise expressions with surds in the denominator, to recognise and calculate with the exact value of surds (irrational numbers) as a mathematical object, rather than only considering its decimal approximation and engage students in using irrational numbers and common surds in calculations in both exact and approximate form
- teach students to use the Euclidean division algorithm to find the greatest common divisor (also called the highest common factor) of two natural numbers and to express relations between sets (membership, complement, intersection, union, subset) for up to three sets, illustrating this in a variety of ways
- teach students to identify and represent linear, quadratic and exponential functions by table, rule and graph using independent and dependent variables, domain and range and to distinguish between, use and interpret these types of functions by testing for constant first or second difference or constant ratio between consecutive terms and modelling a range of contexts
- teach students to solve equations of the form f(x) = k, where k is a real and simultaneous linear equation in two variables using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods and to recognise and explain the roles of the relevant constants in the relationships f(x) = a x + c,

with reference to gradient and y axis intercept, $f(x) = a(x + b)^2 + c$ and $f(x) = cax$

- teach students to use perspective, isometric drawings, nets and computer-generated images to make representations and to recognise and describe boundaries, surfaces and interiors of common plane and 3-D shapes, including cylinders, spheres, cones, prisms and polyhedra and to explore the effect of changing the scale of one characteristic of 2-D and 3-D shapes on related characteristics
- teach students to recognise features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use associated angle properties and engage them in exploring properties of spheres.
- teach students to calculate constant rates and interpret and use mensuration formulas for calculating the perimeter, surface area and volume of familiar 2-D and 3-D shapes and simple composites and engage them in estimating and measuring length, area, surface area, mass, volume, capacity, rates and angle (degrees and radians), selecting and using appropriate units, converting between units as required and deciding on acceptable levels of error in a given situation
- teach students to apply isometric and similarity transformations of geometric shapes in the plane and to identify points that are invariant under a given transformation and to use latitude and longitude to locate places on the Earth's surface and measure distances between places using great circles.
- teach students to use Pythagoras' theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles
- teach students to estimate probabilities based on data, assign and justify subjective probabilities in familiar situations and calculate probabilities for complementary, mutually exclusive, and compound events (defined using *and*, *or* and *not*) and engage them in listing event spaces (for up to three events) by lists, grids, tree diagrams, Venn diagrams and Karnaugh maps (two-way tables) and classifying events as dependent or independent.
- teach students to comprehend the difference between a population and a sample, to generate data using surveys, experiments and sampling procedures, to calculate summary statistics for centrality (mode, median and mean), spread (box plot, inter-quartile range, outliers) and association (by-eye estimation of the line of best fit from a scatter plot) and to distinguish informally between association and causal relationship in bi-variate data, and make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables
- engage students in choosing, using and developing mathematical models and procedures to investigate and solve problems set in a wide range of practical, theoretical and historical contexts and in formulating and testing conjectures, generalisations and arguments in natural language and symbolic form generalising from one situation to another; then to investigate further by changing the initial constraints or other boundary conditions and judging the reasonableness of their results
- engage students in selecting and using technology in various combinations to assist in inquiry, to manipulate and represent data, to analyse functions and carry out symbolic manipulation and in using geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

For students achieving above the expected level, teachers should refer to the appropriate VELS standards and progression points

For students needing additional assistance, teachers should refer to earlier VELS standards and progression points

Assessment

Effective teachers continually monitor and track the progress of individual students.

At the **beginning of the year**, effective teachers:

- use data including: VELS teacher judgements, VCAA on demand testing and year 9 NAPLAN results to understand the starting point for each student
- administer and analyse VCAA On Demand Adaptive Testing – a range of computer-based assessments which identify a student's achievement level. It is recommended that teachers use this tool to identify the

spread of achievement within the class and then to use the progress test to gain more detailed information about individual students.

For students achieving below the expected level:

- administer and analyse Fractions and Decimals Online Interview
- administer the assessment materials to assess students' multiplicative thinking. Scaffolding Numeracy in the Middle Years, analyse the assessment outcomes using Learning and Assessment Framework for Multiplicative Thinking and plan for future learning by using Learning plans.

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual students against VELS Mathematics standards and progression points
- administer and analyse, for identified students, the Assessment for Common Misunderstandings. The tools comprise a number of easy to administer, practical assessment tasks designed to address key areas of Number.
    - (LEVEL 6 – Generalising, skills and strategies to support equivalence, recognition of number properties and patterns, and the use of algebraic text without which it is impossible to engage with broader curricula expectations at this level)
    - (LEVEL 5 – Proportional reasoning, extending what is known about multiplication and division beyond rule-based procedures to solve problems involving fractions, decimals, percent, ratio, rate and proportion)
    - (LEVEL 4 – Partitioning, the missing link in building common fraction and decimal knowledge and confidence)
- administer and analyse VCAA On Demand Progress tests which are linear tests designed to measure outcomes against the VELS
- regularly observe students working mathematically across all dimensions within the class context, particularly during independent mathematics and document progress against the VELS standards and progression points
- provide students with regular opportunities for self assessment and self reflection
- use the VCAA mathematics assessment maps to help assess student work using the VELS to support consistent, on-balance judgments about student achievement.

At the **end of each semester**, effective teachers:

- integrate evidence collected throughout the semester to make on balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective mathematics teachers:

- demonstrate capabilities as described in the e^5 Instructional Model
- dedicate as a minimum four hours weekly to explicit mathematics teaching
- use a range of flexible student groupings ensuring appropriate level of differentiated teacher support: whole class focus, small groups, independent activities and whole class reflection and analysis
- organise learning spaces to enable differentiated teaching and collaborative learning
- plan and design sessions, making connections to mathematical ideas from previous sessions or experiences, different parts of mathematics or contexts
- structure purposeful, authentic mathematical tasks that engage students to envisage different possibilities, use a range of strategies allowing diverse products to emerge, and encourage higher order thinking skills
- develop student mathematical understanding through teacher strategic questioning and feedback and student explanation of reasoning and methods, as well as using multiple representations to make content

accessible to all

- provide independent mathematics time to enable student to practise what they know and to act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by introducing new terms and symbols meaningfully and expect and encourage correct use, making connections between language, symbols and materials
- provide opportunities and resources that enable students to manipulate concrete materials to construct their own understandings and use these resources in the application of concepts and skills.

Recommended teaching strategies

Effective numeracy teachers:

- differentiate instruction according to student need
- explicitly teach students strategies to approach mathematical problems,
- select appropriate teaching strategies including Classroom Activities, following analysis of the Fractions and Decimal Online Interview
- engage students in discussion, reflection and active construction throughout sessions, to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of scaffolding practices to support teachers to make more informed decisions about how they will meet the learning needs of all students in the most appropriate way possible
- select from the twelve scaffolding practices and use appropriate to purpose throughout the five phases of instruction as described in the e^5 Instructional Model.

Considerations for materials selection

Effective mathematics teachers select a range of materials that

- are based on the students' developmental stage and the mathematics being explored
- are appropriate to the learning context and are socially and culturally inclusive.

